White Paper

Differential Ray Tracing in CODE V

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Abstract

Differential ray tracing is a well-known technique in geometrical optics. There are a variety of applications for this technique. In this paper, the definition for differential ray tracing is given and some applications for differential ray tracing in CODE V are presented. General methods for computing differential ray information are briefly discussed.

Definition of Differential Ray Tracing

Consider some general optical system. For this system, locate a Cartesian coordinate system in the object space and one in the image space (in this paper, image space quantities are distinguished by a prime: ´). Now consider some ray through the system, such as the one shown in Figure 1.

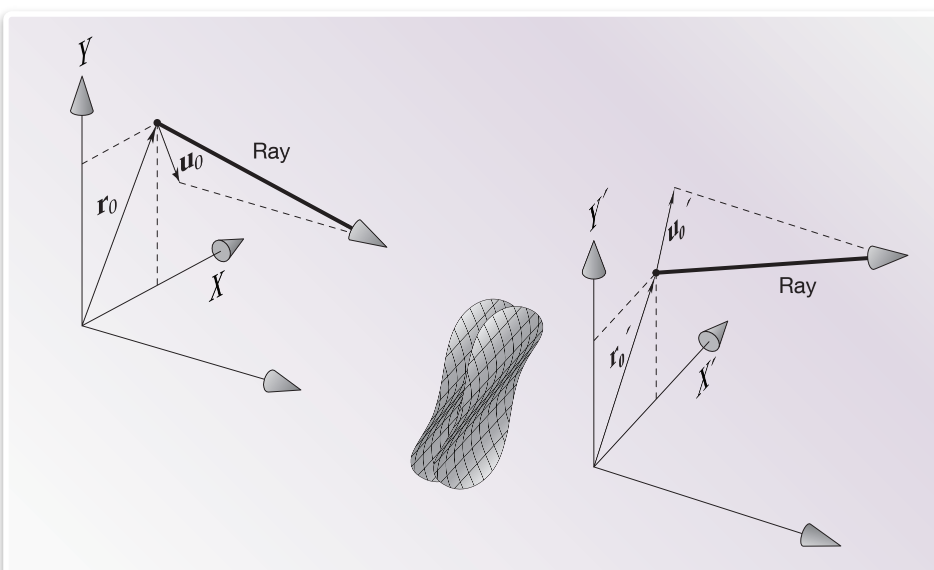

Figure 1: This figure illustrates a ray in the object space and image space of a general optical system.

In that figure, r_0 represents the initial position of the ray (on the plane Z = 0, say), while u_0 represents the initial ray direction. Similar quantities are defined for the image space. In general, one can consider the final ray configuration to be a function of the initial ray configuration:

$$r'(r,u), \qquad u'(r,u)$$

These functions can be quite complicated and generally cannot be determined in closed form for anything but the simplest of optical systems. Note, however, that it is generally possible to determine the output position and direction of a ray for any given input position and direction. That is, given an input ray (with initial position and direction of r_0 and u_0, for example), one can trace the ray through the system to determine its output position and direction (r'_0 and u'_0, for example). While this discrete ray information allows one to perform a variety of analyses on an optical system, for some computations (such as the ones described below), it is convenient to have not just discrete ray data, but also to include *differential ray* data.

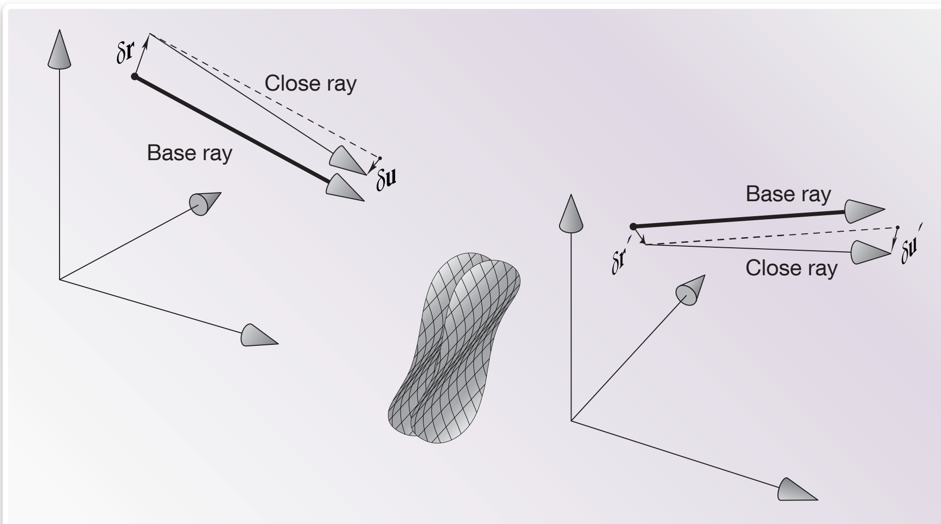

Figure 2: This figure illustrates a ray that is closely spaced about the base ray. Differential ray data about the base ray can be used to determine the change in the final configuration from the base ray ($\delta\mathbf{r}'$ and $\delta\mathbf{u}'$) given the change in the initial configuration ($\delta\mathbf{r}$ and $\delta\mathbf{u}$).

To understand the differential ray data, consider the ray shown in Figure 1. The differential ray data for this ray allows one to determine (approximately) the configuration of all closely spaced rays about this ray (which is henceforth referred to as the base ray). This situation is illustrated in Figure 2. A ray that is closely spaced about this base ray is also shown. The change in position and direction of the close ray from the base ray in the object space is labeled as δr and δu, respectively, and for the image space the change is labeled as $\delta r'$ and $\delta u'$. By including differential ray data for the base ray, the configuration of closely spaced rays are determined by means of a linear approximation:

$$\mathbf{r}'(\mathbf{r},\mathbf{u}) \cong \mathbf{r}'_0 + \mathbf{A}\delta\mathbf{r} + \mathbf{B}\delta\mathbf{u} \;\;\rightarrow\;\; \delta\mathbf{r}' \cong \mathbf{A}\delta\mathbf{r} + \mathbf{B}\delta\mathbf{u}$$

$$\mathbf{u}'(\mathbf{r},\mathbf{u}) \cong \mathbf{u}'_0 + \mathbf{C}\delta\mathbf{r} + \mathbf{D}\delta\mathbf{u} \;\;\rightarrow\;\; \delta\mathbf{u}' \cong \mathbf{C}\delta\mathbf{r} + \mathbf{D}\delta\mathbf{u}$$

The coefficients A, B, C, and D represent the differential ray information about the given base ray. Note that in general it takes two quantities to specify the position of a ray (e.g., the *X* and *Y* coordinates on some plane), and two to specify the direction, so that A, B, C, and D are generally not scalars but are themselves 2×2 matrices.

With the differential ray information, one knows not just what a single ray is doing, but there is now a region about the base ray for which one knows how rays behave (without having to trace additional rays). This is illustrated schematically in Figure 3.

Also note that the definition of differential ray information is commonly expanded to include quantities that can be easily derived from A, B, C, and D, such as the change in the optical length along a close ray from that of the associated base ray. Another way in which the definition of differential ray tracing has broadened is also discussed in the following subsection on tolerancing.

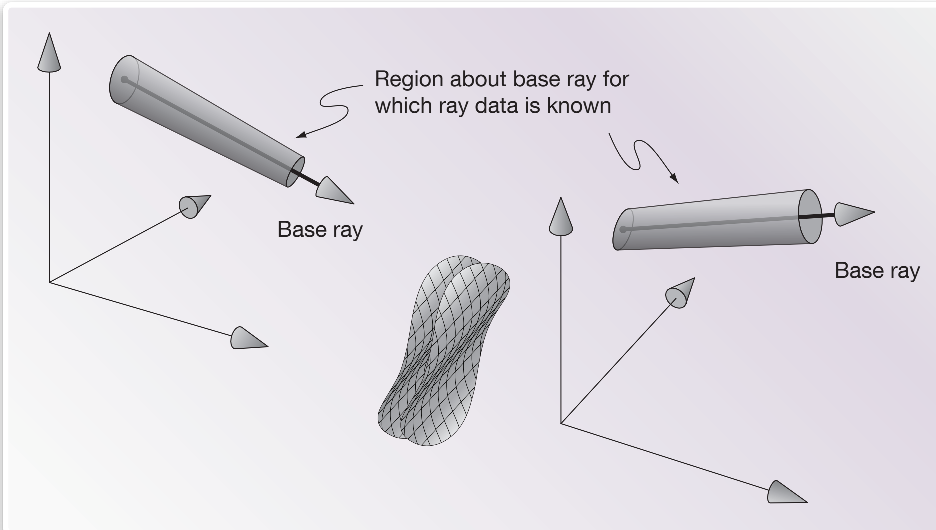

Figure 3: With differential ray information, one knows how rays near the base ray behave without having to trace additional rays.

The next section contains a discussion of three applications where differential ray information is used to facilitate computations. This is followed by a short description of methods for computing this differential ray information (i.e., the coefficients A, B, C, and D).

Applications of Differential Ray Tracing

First order properties

Differential ray data is commonly used to compute the so-called first-order properties of an optical system (in fact, the first-order properties are generally defined in terms of the differential ray data). For rotationally symmetric optical systems, the base ray is conventionally taken to correspond with the ray straight down the axis of the system. In this case, the symmetry of the system allows the 2×2 matrices A, B, C, and D that represent the differential ray information to be reduced to scalar quantities, *A*, *B*, *C*, and *D*.

As shown in Figure 4, differential ray information can be used to compute familiar first-order quantities such as the paraxial image location and magnification.

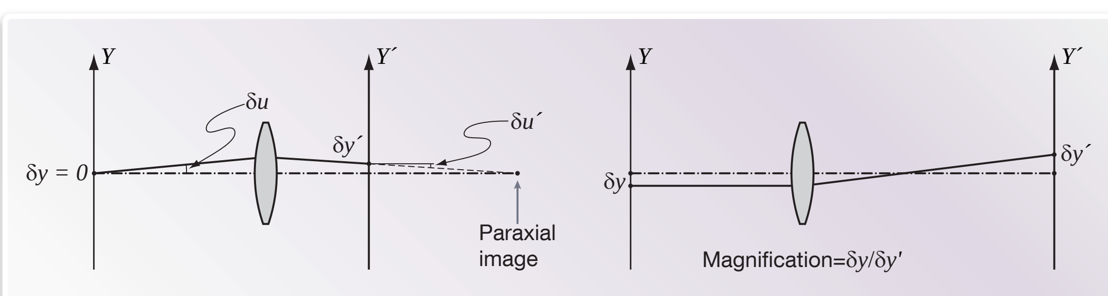

Figure 4: Schematic illustration of the use of differential ray information to determine the paraxial image location and the magnification in an optical system.

Differential ray information about the axial ray is used similarly to determine the locations of the so-called Cardinal points in an optical system, and to determine the approximate size of the illuminated patch on each surface of an optical system.

The situation for systems without symmetry is more complicated in detail but it is conceptually similar: a ray from the center of the object through the center of the aperture stop is chosen as the base ray, and differential ray data about this ray is computed. Concepts such as image location and focal points generalize but can still be computed from differential ray information[1].

Gaussian beam propagation

The problem of modeling the propagation of Gaussian beams lies in the realm of physical optics. Nonetheless, geometrical optics — and in particular differential ray data — can be used to model the propagation of Gaussian beams. Consider some Gaussian beam that is input into a symmetric system along the axis and say this beam starts with width w and wavefront radius, R. After propagation through the system, the beam has width w' and wavefront radius R'. This is illustrated in Figure 5. Further, say that one has computed the differential ray information about the base ray that corresponds with the axis of the system. This differential ray information can be used to determine the output beam parameters as a function of the input beam parameters[2].

This is commonly done by defining a complex radius of curvature, q, as follows:

$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi w^2}$$

where λ is the wavelength of light and i is the square root of negative one. The complex radius of curvature of the Gaussian after propagation through the system can be determined from the input Gaussian and the differential ray information as follows:

$$q' = \frac{Aq+B}{Cq+D}$$

The width of the output Gaussian and its wavefront radius of curvature then can be determined from the complex radius of curvature.

Finally, note that when the Gaussian is not propagating along the axis, the computation is more involved, but still only involves the differential ray information about the central ray of the Gaussian[3].

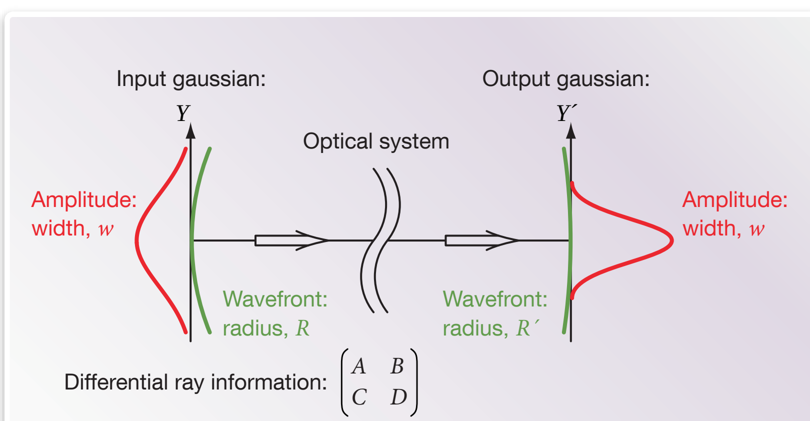

Figure 5: A Gaussian beam propagating through a rotationally symmetric system.

Tolerancing

As part of the process of assigning manufacturing tolerances to an optical system, it is necessary to know how rays change when an optical system is perturbed. In TOR, CODE V's fast tolerancing feature, tolerances are generally determined by considering changes to wavefronts. Thus, it is the change in the optical path length (OPL) along a ray from a given object point to a given point in the exit pupil that is of interest, and not the change of ray position at the image. By collecting these changes in OPL for a variety of rays when an optical system is perturbed, the change in image quality can be determined. Based on these changes, acceptable manufacturing tolerances are then assigned.

At first glance, it appears that differential ray information may be useful for this process. For example, consider Figure 6. This figure shows an optical system in which one of the surfaces has been perturbed (the curvature has gone from c to $c+\delta c$, say). A ray that starts at a given point on the object and passes through a given point on the exit pupil is shown both for the unperturbed and perturbed system. While the picture might suggest that differential ray information will be useful in determining the change in optical path length, it turns out that on account of Fermat's principle, the differential ray data as defined in this document is not necessary[4]. While the details are somewhat involved, it turns out that a good estimate for the change in OPL can be determined simply from the change in height of the surface (this is labeled h in Figure 6) and the angle of incidence and refraction for the unperturbed ray:

$$\delta \mathrm{OPL} = (n\cos\theta - n'\cos\theta')h$$

where n (n') is the refractive index before (after) the surface. While, strictly speaking, differential ray information is not used for this computation, this approximate form for the change in OPL, which is an integral part of the CODE V tolerancing[5], is often thought of as falling under the rubric of differential ray tracing.

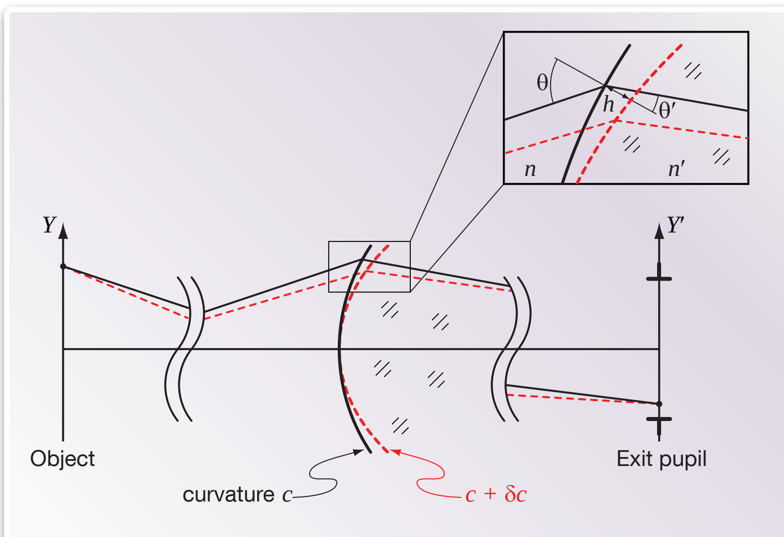

Figure 6: Schematic illustration of a system with a perturbation. A ray through the perturbed system and one through the unperturbed system are shown. These rays both start from the same object point and pass through the same point in the exit pupil.

Note that there is a connection between the process of optimization and tolerancing. As such, differential ray information also is used during optimization for some CODE V merit functions[6]. For example, the use of differential ray tracing allows CODE V to realize significant gains in efficiency for optimization with MTF-based merit functions[7].

Computing Differential Ray Information

The simplest way to compute differential ray information is to trace a set of real rays that are closely spaced about the base ray and use finite differencing to estimate the differential ray information. This is akin to computing the derivative of a function numerically via finite differencing, which is illustrated in Figure 7.

For a symmetric system with the base ray going straight down the axis, two additional closely spaced rays are required to determine estimates for the differential ray information. In the most general case, four additional rays are required (although similarly to conventional finite differencing, more rays can be used to increase the accuracy of the estimates).

While finite differencing provides the simplest means to compute differential ray information, it requires that an appropriate derivative increment (i.e., the analog of δx in Figure 7) be specified. This difficulty can be overcome for many optical systems by computing the differential ray information exactly[8]. These methods are generally more computationally efficient but require many more lines of specialized code (e.g., different code is needed to compute the differential ray information across an aspherical surface than is needed for a spherical surface).

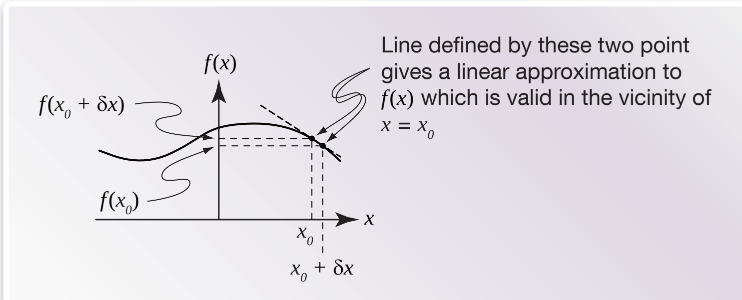

Figure 7: Schematic illustration of the concept of finite differencing.

Concluding Remarks

The study of general differential ray tracing goes back well over one hundred years[9] and the techniques involved in computing differential ray information are very well understood. In addition to the works cited above, numerous contributions to this field have been made by others[10]. These techniques are well understood by the scientists at Synopsys and they have been incorporated into CODE V for the applications described above, as well as for other applications in which differential ray information provides computational advantages.

References

[1] Bryan D. Stone and G.W. Forbes, "Characterization of first-order optical properties for asymmetric systems," *J. Opt. Soc. Am. A* 9, 478-489 (1992).

[2] Anthony E. Siegman, *Lasers* (University Science Books, Mill Valley, CA, 1986), Sec. 20.2.

[3] See Arnaud and Kogelnik, "Gaussian Light Beams with General Astigmatism", *Applied Optics*, Vol. 8, No. 8, 1969 and Suematsu and Fukinuki, "Matrix Theory of Light Beam Waveguides", *Bull. Tokyo Inst. Tech*, Number 88, 1968.

[4] This is described, for example, in Bryan D. Stone, "Perturbations of optical systems," *J. Opt. Soc. Am. A* 14, 2837-2849 (1997).

[5] See, for example, H.H. Hopkins and H.J. Tiziani, *Brit. J. Appl. Phys.*, 17, 33 (1966) or M. Rimmer *Applied Optics*, Vol. 9 Issue 3 Page 533 (March 1970).

[6] For a description of the use of differential ray tracing in optimization, see Donald P. Feder, "Differentiation of ray-tracing equations with respect to construction parameters of rotationally symmetric optics," *J. Opt. Soc. Am.* 58, 1494-1505 (1968).

[7] M.P. Rimmer, T.J. Bruegge and T.G. Kuper, "MTF Optimization in Lens Design," *SPIE*, Vol. 1354, 1990, p. 83.

[8] For the mechanics of differential ray tracing through homogeneous media, see, for example, A. Cox, *A System of Optical Design* (Focal, London, 1964), pp. 112-121. For inhomogeneous media, see Bryan D. Stone and G.W. Forbes, "Differential ray tracing in inhomogeneous media," *J. Opt. Soc. Am. A* 14, 2824-2836 (1997).

[9] For example, these concepts are discussed by J. Larmor, "The characteristics of an asymmetric optical combination," *Proc. Lond. Math. Soc.* 20, 181-194 (1889).

[10] As examples, see T. Smith, "The primordial coefficients of asymmetrical lenses," *Trans. Opt. Soc.* 29, 167-178 (1928); R.K. Luneburg, *Mathematical Theory of Optics* (U. California Press, Los Angeles, 1964) Sec. 36; M. Herzberger, First-order laws in asymmetrical optical systems — part I. The image of a given congruence: fundamental conceptions," *J. Opt. Soc. Am.* 26, 254-359 (1936); P.J. Sands, "First-order optics of the general optical system," *J. Opt. Soc. Am.* 62, 369-372 (1972).